**Mathematics Courses List**

**Contexts**

**I. Analysis and Differential Equations:…………………………………2**

1. Mathematical Analysis A1~A3;

2. Differential Equations I;

3. Real Analysis;

4. Complex Analysis (H);

5. Functional Analysis;

\*6. Advanced Real Analysis;

\*7. Differential Equations II (H);

\*8. Harmonic Analysis;

\*9. Nonlinear PDEs in Fluid Dynamics;

\*10. Second Order Elliptic PDEs;

\*11. Nonlinear Dispersive PDEs;

**II. Probability and Stochastic Processes:……………………….………5**

1. Probability Theory;

\*2. Advanced Probability Theory;

\*3. Advanced Stochastic Processes;

\*4. Probability Limiting Theory;

\*5. Topics in Stochastic Analysis;

\*6. Martingale Theory and Stochastic Calculus.

**III. Geometry and Topology:………………………………………………7**

1. Differential Geometry;

2. Topology;

\*3. Differential Manifolds;

\*4. Riemann Geometry.

**IV. Algebra:…………………………………………………………………8**

1. Linear Algebra A1, A2;

2. Introduction to Algebra;

3. Abstract Algebra.

Note: \* means this is a graduate level course

**I. Analysis and Differential Equations:**

**1. Mathematical Analysis A1~A3**

**Instructor: Guangbin Ren (A1), Jihuai Shi (A2, A3); Grade: 89, 99, 96/100.**

Textbook: Gengzhe Chang, Jihuai Shi: *A Course in Mathematical Analysis*.

Contents: **A1: One-Variable Analysis:**

1. Limit of Sequences;

2. Continuity and Differentiation of One-variable Functions;

3. Taylor’s Formula;

4. Riemann’s Integration and Lebesgue’s Theorem, Stirling’s Formula;

**A2: Multi-Variable Analysis:**

1. Topology of **R**d;

2. Continuity and Differentiation of Multi-variable functions;

3. Inverse and Implicit Mapping Theorem;

4. Multi-variable Integral (Green’s Formula, Stokes’ Formula, An intro. to Field Theory);

**A3: Series and Generalized Integral:**

1. Convergence of Number Series, Function Series including Power Series;

2. Convergence of Generalized Integral;

3. Fourier Series and Its Convergence;

4. Convergence of Generalized Integral with Parameters.

**2. Differential Equation I**

**Instructor: Benjin Xuan; Grade: 92/100.**

Textbook: [1] Tongren Ding, Chengzhi Li: *Textbooks of Ordinary Differential Equations*;

[2] Lawrence C. Evans: *Partial Differential Equations*, Ch. 1, 2, 4.1.

Contents: 1. Methods of solving to some ODEs;

2. Picard’s Iteration: the Existence and the Uniqueness of ODE;

3. Extension of Solutions of ODEs;

4. Solving ODE Systems;

5. Introduction to the Planar Dynamical Systems determined by ODEs;

6. Transport Equation;

7. Laplacian Equation: Fundamental Solutions, Green’s Function, Gradient Estimates;

8. Heat Equation: Fourier Transform Method;

9. Classical Solution to Wave Equation;

10. Separation of Variables.

**3. Real Analysis**

**Instructor: Lifeng Zhao; Grade: 97/100.**

Textbook: [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 1, 2, 3, 6.

[2] Minqiang Zhou: *Functions of Real Variables*, L^p space part.

Contents: 1. Lebesgue Measure Theory in **R**d;

2. Lebesgue Integration Theory and Convergence Theorems, Lp space;

3. Differentiation of Functions on **R**: Lebesgue’s Density, Functions of Bounded Variation, Absolute and Lipschitz Functions.

4. Abstract Measure Theory: Extension Theorem, Lebesgue-Stieltjes Measure.

**4. Complex Analysis (H)**

**Instructor: Luo Luo; Grade: 84/100.**

Textbook: Jihuai Shi, Taishun Liu: *Functions of Complex Variables*.

Contents: 1. Topology of **C**;

2. Cauchy-Riemann Equations and Holomorphic Functions;

3. Cauchy-Pompeiu Integral Formula;

4. Complex Series: Taylor’s and Laurent’s Expansion;

5. Memomorphic Functions and Residue Formulae;

6. Holomorphic Extension;

7. An Introduction to Riemann’s Mapping Theorem.

**5. Functional Analysis**

**Instructor: Yi Wang; Grade: 85/100**

Textbook: Kung-ching Chang, Yuanqu Lin: *Lecture Notes on Functional Analysis*.

Contents: 1. Metric Space and Contraction Principle;

2. Linear Operators: Riesz Representation Thm, Open Mapping Thm, Hahn-Banach Thm.

3. Weak and Weak-\* Convergence of Linear Operator Series;

4. Spectrum of Compact Linear Operators: Riesz-Schauder Theory.

**6. Advanced Real Analysis**

**Instructor: Lifeng Zhao; Grade: 97/100**

Textbook: [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 6;

[2] Elias M. Stein, R. Shakarchi: *Functional Analysis*, Chapter 1, 2, 3, 8.

Contents: 1. Abstract Measure Theory: Extension Theorems and Applications, Signed Measure and Radon-Nikodym Theorem;

2. Lp space and Inequalities;

3. Riesz-Thorin Interpolation, Hilbert Transform, Hardy-Littlewood Maximal Function;

4. Distribution: Basic Calculation, Tempered Distribution, Homogeneous Distribution, Parametrices of Linear Partial Differential Operators;

5. An Introduction to Oscillatory Integrals.

**7. Differential Equation II (H)**

**Instructor: Lifeng Zhao; Grade: 90/100**

Textbook: Lawrence C. Evans: *Partial Differential Equations*, Chapter 5, 6, 7, 8.1, 8.6, 9.4.

Contents: 1. Sobolev Spaces **Wk,p(U)** and **Hs(Rd)**;

2. Elliptic Equations: Weak Solutions, Maximum Principle, Fourier Expansion;

3. Parabolic Equ. : Galerkin’s Method, Theory of Weak Solutions, Maximum Principle;

4. Hyperbolic Equ. : Galerkin’s Method, Existence and Regularity to Weak Solutions;

5. Vanishing Viscosity Method for 1st Order Hyperbolic Systems;

6. 3D Cubic Nonlinear Schrodinger’s Equation and Its Strichartz Estimates;

7. Conservation Law: Euler-Lagrange Equations, Nother’s Theorem, Pohozaev’s Identity for △*u*=*u*|*u*|*p*-1 with zero boundary data.

**8. Harmonic Analysis**

**Instructor: Guangbin Ren; Grade: 95/100**

Textbook: [1] Javier Duoandikoetxea: *Fourier Analysis*, Chapter 1, 2;

[2] Elias M. Stein: *Singular Integrals and Differentiability of Functions*, Chapter 2, 3;

[3] C. Muscalu, W. Schlag: *Classical and Multilinear Harmonic Analysis*, Chapter 8;

[4] M. Ruzhansky: *Pseudo-Differential Operators and Symmetries,* Chapter 6, 7.

[5] M. Wong: *Wavelet Transforms and Localization Operators*, Section 6-9, 17-18.

Contents: 1. Fourier Series and Fourier Transforms, Radon Transforms;

2. Hardy-Littlewood Maximal Operators;

3. Singular Integrals: Calderon-Zygmund Theory;

4. Littlewood-Paley Theory;

5. Representation Theory of Compact Groups(including Peter-Weyl Theorem);

6. Wavelet Transform;

7. Representation Theory of Affined Group and Weyl-Heisenberg Group.

**9. Nonlinear PDEs in Fluid Dynamics**

**Instructor: Lifeng Zhao; Grade: 96/100**

Textbook: [1] Andrew J. Majda, Andrea L. Bertozzi: *Vorticity and Incompressible Flows*;

[2] Changxing Miao, Jiahong Wu, Zhifei Zhang: *Littlewood-Paley Theory and Its Applications to PDEs in Fluid Dyamics*.

[3] **J. Bedrossian, N. Masmoudi: *Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, 2015.**

Contents: 1. Sobolev Inequalities, Calderon-Zygmund Singular Integral;

2. Littlewood-Paley theory, Besov Spaces and Paraproduct Decomposition;

3. Conservation Laws in 2D, 3D Euler’s Equation;

4. Local Existence of the solution to 2D Euler’s Equations;

5. Beale-Kato-Majda Criterion and Global Existence of 2D Euler’s Equation;

6. Leray-Hopf Weak Solutions;

7. Semi-linear Methods for Navier-Stokes and Fujita-Kato;

8. Strong solution to NSE: Mild solution and weak-strong uniqueness;

9. Spectral stability and Lyapunov stability of Euler’s equation;

10. 2D Inviscid Planar Shear Flows and Rayleigh’s theorem;

11. Arnold’s Nonlinear Stability Theorem for Shear Flows in a Channel;

12. Mixing, Trade regularity for decaying;

13. Zillinger’s theorem;

**14. *Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, J. Bedrossian, N. Masmoudi, *Publications mathématiques de l'IHÉS*.**

**10. Second Order Elliptic PDEs**

**Instructor: Hong Zhang; Grade: 90/100**

Textbook: David Gilbarg, Neil S. Trudinger: *Second Order Elliptic PDEs,* Chapter 1-9.

Contents: 1. Fundamental Solution to Laplacian, Gradient Estimates, Maximum Principle;

2. Newton Potential and Its Holder Estimates;

3. Schauder Estimates;

4. Weak Solution: Sobolev Spaces Theory, Existence and Regularity of Weak Solutions;

5. Boundedness: Di Giorgi-Moser Iteration;

6. Strong Solution: Lp Theory and Calderon-Zygmund Theorem.

**11. Nonlinear Dispersive PDEs**

**Instructor: Lifeng Zhao; Grade: 97/100**

Textbook:

[1] Terence Tao: *Local and global analysis of nonlinear dispersive and wave equations*;

[2] Christopher D. Sogge: *Lectures on Nonlinear Wave Equations, 2nd edition;*

[3] Jonathan Luk: *Lecture Notes on Nonlinear Wave Equations;*

[4] Sergiu Klainerman: *Lecture Notes of Analysis in Princeton, 2011*;

[5] Pierre Germain, Fabio Pusateri, Frederic Rousset: *Asymptotic Stability of solutions for mKdV, Advances in Mathematics, 2016;*

[6] Changxing Miao, Bo Zhang: *Harmonic Analysis Methods in PDEs*.

Contents:

1. Prerequisites: Distribution, Oscillatory Integrals;

2. Derivation of Dispersive PDEs: Hamiltonian, Euler-Lagrange Method, Water wave eq;

3. Linear Wave Equ: Fundamental Solution, Klainerman-Sobolev Inequality, Strichartz Estimates of Wave Equation;

4. Linear Schrodinger Equ: Strichartz Estimates, Christ-Kiselev Lemma, Keel-Tao Endpoint Estimates.

5. Local Well-posedness of NLS in subcritical and critical spaces;

6. Conservation Law: Conservations Laws for NLS, Morawetz Estimates, Global Well-posedness of NLS, Decay and Scattering of NLS and NLW.

7. Asymptotic Stability of mKdV.

**II. Probability and Stochastic Processes:**

**1. Probability Theory**

**Instructor: Dang-zheng Liu; Grade: 88/100**

Textbook: G. Grimmett, D. Stirzaker: *Probability and Random Process*, Chapter 1~5, 7.

Contents: 1. Probability Measure and Random Variable;

2. Discrete Random Variables;

3. Continuous Random Variables;

4. Generating Function Method;

5. Characteristic Functions;

6. Convergence of Random Variables;

7. CLT(iid case, Lindeberg-Feller) and LLN.

**2. Advanced Probability Theory**

**Instructor: Lijun Bo; Grade: 91/100**

Textbook: Kai-lai Chung: *A Course in Probability*.

Contents: 1. Probability Measure Theory;

2. Random Variable and Distribution;

3. Expectation and Conditional Expectation (w. r. t. Sigma-Algebra);

4. Convergence and Tightness of Random Variable Sequences;

5. CLT and LLN.

**3. Advanced Stochastic Processes**

**Instructor: Lijun Bo; Grade: 97/100**

Textbook: I. Karatzas, S. E. Shreve: *Brownian Motion and Stochastic Calculus*, GTM113, Ch 1~3.

Contents: 1. Basic Definitions about Stochastic Processes;

2. Martingale Theory and Its Convergence Theorems;

3. Brownian Motion: Wiener Measure, Reflection Principle, Markov Property, Donsker Theorem.

**4. Probability Limiting Theory**

**Instructor: Zhishui Hu; Grade: 93/100**

Textbook: [1] Rick Durrett: *Probability: Theory and Examples*, 4th edition, 2010. Ch 2, 3, 5;

[2] Zhengyan Lin, etc: Basis of Limiting Theory.

[3] Olav Kallenberg: *Foundations of Modern Probability*, 1997, Ch 4.

Contents: 1. Weak and Strong Law of Large Numbers;

2. Three Series Theorem;

3. Law of Iterated Logarithm by Hartman-Witner;

4. Central Limit Theorem (iid, Lindberg-Feller) and Karamata Slow-Variation Theorem;

5. Stable Law, Poisson Convergence, Infinite Divisible Distribution;

6. Discrete Martingale Theory;

7. Skorohod’s Topology and Probability Theory in Polish Spaces.

**5. Topics in Stochastic Analysis**

**Instructor: Elton P. Hsu, Dang-zheng Liu, Ran Wang; Grade: 97/100**

Textbook: [1] Terence Tao: *Topics in Random Matrix Theory*;

[2] Louis Chen: *An Introduction to Stein’s Method*;

[3] Varadhan: *Lecture Notes on Large Deviation Theory*.

Contents: **Part 1: An Introduction to Random Matrix Theory:**

1. Examples of random matrices and problems;  
2. Wigner matrices and semicircular law;  
3. GOE, GUE and Tracy-Widom distribution.

**Part 2: An Introduction to Stein’s Method:**

1. Gaussian Measures;

2. Functional inequalities (Poincare, log-Sobolev, and Beckner)  
3. Stein’s method and Central Limit Theorem.

**Part 3: An introduction to Large Deviation Theory:**

1. Large Deviation of i.i.d. sequences, Markov Chain, Brownian Motion;

2. Sanov’s Theorem and Schilder’s Theorem.

**6. Martingale Theory and Stochastic Integrals**

**Instructor: Jianliang Zhai; Grade: 96/100**

Textbook: Elton P. Hsu: *Lecture Notes on Martingale Theory and Stochastic Integrals*.

Contents:

1. Martingale Theory;

2. Brownian Motion: Wiener Measure, Markov Property, Reflection Principle, Quadratic Variation.

3. Stochastic Integration and Ito’s Formula;

3.1 Stochastic Integrals w.r.t Brownian Motion;

3.2 Stochastic Integrals w.r.t Continuous Local Martingale;

3.3 Ito’s Formula of L2 semi-martingale;

3.4 Stratonovich’s Stochastic Integral.

4. Applications of Ito’s Formula;

4.1 Levy’s Characterization of Brownian Motion;

4.2 Exponential Martingale and Uniform Integrability;

4.3 Girsanov’s Theorem;

4.4 B-D-G inequality;

4.5 Representation Theorem of Martingales;

4.6 Reflective Brownian Motion and Brownian Bridge.

5. Stochastic Differential Equations and Its Applications to Financial Mathematics.

**III. Geometry and Topology:**

**1. Differential Geometry Instructor: Xiaowei Xu; Grade: 87/100**

Textbook: Jiagui Peng, Qing Chen: *Differential Geometry*.

Contents: 1. Local Theory of 2D and 3D Curves;

2. Local Theory of Surfaces (1st, 2nd fundamental form, Weingarten Transform, Gauss Curvature);

3. Frames and the Fundamental Theorem of Theory of Surfaces;

4. Intrinsic Geometry of Surfaces (Co-variant Differentiation, Geodesic Curve, Gauss-Bonnet Formula, Laplacian of a Surface, Riemann’s Metric);

5. Global Properties of Surfaces (Gauss-Bonnet Formula, Gauss Mapping for Compact Surfaces, Convex Surfaces).

**2. Topology Instructor: Bailin Song; Grade: 80/100**

Textbook: [1] Chengye You: *General Topology*, Peking University;

[2] Allen Hatcher: *Algebraic Topology*.

Contents: 1. Basic Properties: Toplogical Spaces, Compactness, Connectness and Path-connectness, Homeomorphisms;

2. Classification of Closed Surfaces in 3D.

3. Homotopy and Fundamental Group, Van-Kampen Theorem;

4. Covering Spaces, Universal Covering Space, Group Actions on Topological Spaces;

5. Simplex Homology, CW complexes;

6. Singular Homology: Calculation, Long and Short Exact Sequences;

7. Tracing Graphs Method and Excision Theorem;

8. Degree of Mappings and Cellular Homology.

**3. Differential Manifolds**

**Instructor: Zuoqin Wang; Grade: 84/100**

Textbook: [1] John M. Lee: *An Introduction to Smooth Manifolds*, GTM218, 2nd edition, Chapter 1-22.

[2] Loring W. Tu: *An Introduction to Manifolds*.

Contents: 1. Smooth Manifolds and Submanifolds;

2. Smooth Mappings and Differentials;

3. Vector Bundles, Tangent and Cotangent Bundles, Tensor Bundles;

4. Vector Fields and Flows;

5. Lie Groups and Their Actions;

6. Differential Forms and Integration;

7. de Rham Cohomology;

8. Riemannian and Symplectic Structures;

9. Other Topics (e. g. Chern-Weil).

**4. Riemann Geometry**

**Instructor: Shiping Liu; Grade: TBA**

Contexts:

1. Riemann Metric;

2. Geodesics, Exponential Maps, Normal Coordinates, Geodesical Completeness and Hopf-Rinow Theorem;

3. Connections (Affine, Levi-Civita), Paralleism, Covariant Derivatives;

4. Curvature (Riemann Curvature Tensor, Sectional and Ricci Curvature);

5. Index Form, Space Forms, Variational Formulae, Jacobi Fields;

6. Candidates for Synthetic Curvature Conditions;

7. Cartan-Hadamard Thm, Bonnet-Meyer Thm, Synge Thm;

8. Comparison Principle;

9. Laplacian, Hessian, Hodge-Laplace Operator;

10. An Introduction to Discrete Geometry and Applications in Graph Theory.

**IV. Algebra**

**1. Linear Algebra A1~A2**

**Instructor: Guangtian Song; Grade: 87, 90/100**

Textbook: Shangzhi Li: *Linear Algebra*.

Contents: 1. Linear Equation System and Matrices;

2. Vector Spaces: Rank, Linear Independence;

3. Determinants (Including Binet-Cauchy Formula);

4. Matrix Theory: Calculation, Rank, Equivalence;

5. Eigenvalue Theory: Eigenvector, Eigenspace, Minimal Polynomial;

6. Jordan’s Form Theory and λ-matrix Theory;

7. Congruence of Symmetric Matrices;

8. Unitary Matrices.

**2. Introduction to Algebra**

**Instructor: Yi-huang Shen; Grade: 91/100**

Textbook: Yi Ouyang, Yihuang Shen: *An Introduction to Algebra*.

Contents: 1. An Introduction to Group, Ring, Field;

2. Elementary Number Theory: Fermat Theorem, Euler’s Theorem, Wilson’s Theorem;

3. Polynomial Rings over **R**;

4. Cylic Groups;

5. Polynomial Rings over Field.

**3. Abstract Algebra**

**Instructor: Mao Sheng; Grade: 87/100**

Textbook: Keqin Feng, Shangzhi Li, Pu Zhang: *An Introduction to Abstract Algebra*.

Contents: 1. Group Theory: Cylic Group, Abel Group, Group Action, Sylow’s Theorem, Free Group, Solvable Group.

2. Ring Theory: Commutative Rings containing unit element.

3. Galois Theory: Field Expansion, Galois Expansion, Galois Theory.